Natural Convection in Stratified Hot Water Storage Tanks

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Abstract

Thermal energy storage is an essential component of solar heating systems. A numerical analysis of thermal stratification in hot water storage tanks suitable for solar water heating applications is presented in this paper. Transient, axi-symmetric, turbulent natural convection in fully and partially charged storage tanks for varying Rayleigh Number in the range of 10^9 to 10^{12} is studied. Mix Number quantifies mixing inside the storage tank whereas Exergy efficiency gives performance based on the energy storage capacity. Numerical results showed good agreement with published experimental results of Abdoly and Rapp [5]. In fully charged storage tank, higher aspect ratio supports stratification. The transient temperature profiles in the bulk fluid reveal formation of stratified layers in fully charged tank and degradation of stratified layers in partially charged tanks due to mixing. Mix Number values, which decrease with increasing aspect ratio, show greater mixing for fully charged tank. The tank charged to ½ capacity showed less mixing compared to those charged to ¼ and ¾. Providing outer tank wall insulation reduces mixing considerably. The results indicate that the heat loss through the walls and insulation to the ambient is dominant compared to thermal diffusion across the thermocline. Exergy efficiency of the storage tanks shows higher values at larger aspect ratio which is attributed due to decrease in mixing in comparison to smaller aspect ratios. It is also seen that with larger aspect ratio, smaller wall thickness and better insulation, mixing can be reduced considerably. The influences of natural convection in the trends are discussed.

Introduction

Natural convection plays a major role in many thermal systems including thermal energy storage systems. Accurate designing of solar heating or cooling thermal energy storage tanks generally requires an account of stratification within the storage tanks, since the overall system performance is significantly affected by the temperature distribution inside the tank. The heat loss from the stored fluid to the ambient decreases the temperature of the fluid near to the tank wall, thereby increasing its density. Due to buoyancy effects, the dense fluid layer moves downward, resulting in development of temperature gradients inside the storage tanks. Natural convective heat transfer and thermal diffusion between the hot and cold fluid are responsible for mixing inside the fully and partially charged storage tank.

The operation of hot water storage tanks for thermal energy storage is classified as static mode and dynamic mode. The static mode is further classified into fully charged and partially charged mode. Dynamic mode of operation includes charging and discharging cycle. In static fully charged mode, the storage tank is initially completely filled with hot water at a constant temperature (T_1) and is subjected to convective heat loss from the tank walls to the ambient. In static partially charged mode, the storage tank is initially charged with different levels of hot and cold water separated by thermocline. In dynamic discharging mode the storage tank is initially filled with warm water, which is drawn from the top of the tank to the load and the returning cold water from the load is charged at the bottom of the storage tank. In dynamic charging mode the storage tank is initially filled with cold water and hot water is charged at same flow rate as the cold water is discharged at the bottom.

Many numerical studies on thermal stratification have been performed showing the resultant temperature distribution inside the storage tank. Hsieh and Lien [1] made a numerical analysis of the turbulent natural convection in enclosures with differentially heated vertical walls showing the best overall performance in terms of mean velocities, temperature and turbulence quantities. Bouhdjar and Harhad [2] developed a two-dimensional model of mixed convection flow in thermal storage tank with varying aspect ratios using finite volume method to determine the transient thermal storage efficiency of thermal energy storage tank. Nelson et al. [3] presented one-dimensional transient heat conduction model to describe the decay of the thermocline in a stratified water tank. The parameters influencing the operation of stratified thermal energy storage for cool storages was examined. Shyu et al. [4] presented theoretical and experimental studies on the stratification decay in stratified tanks and the effects of tank wall thickness and insulation resistances. Abdoly and Rapp [5] neglected tank wall conduction but considered the effects of thermal insulation, and showed that the heat loss through the insulation to the ambient is more than the heat conduction across the thermocline.

The study of literature reveals a lack of information on the mixing inside the tank and the resultant energy capacity of the stored fluid in the heat storage tanks. In the present study, an axi-symmetric transient conjugate heat transfer model that accounts for fully charged and partially charged hot water storage tank is considered. The model also considers heat transfer from the fluid to the ambient and effects of aspect ratio, initial charge in partially charged tank, and also energy loss due to the decay of thermocline. Mix Number and Exergy efficiency defines the performance of the hot water storage tanks.

Physical Model and Governing Equations

Numerical investigation is carried on cylindrical fibreglass hot water storage tank with aspect ratio (length/diameter) ranging from 1 to 4 is shown in Figure. 2.

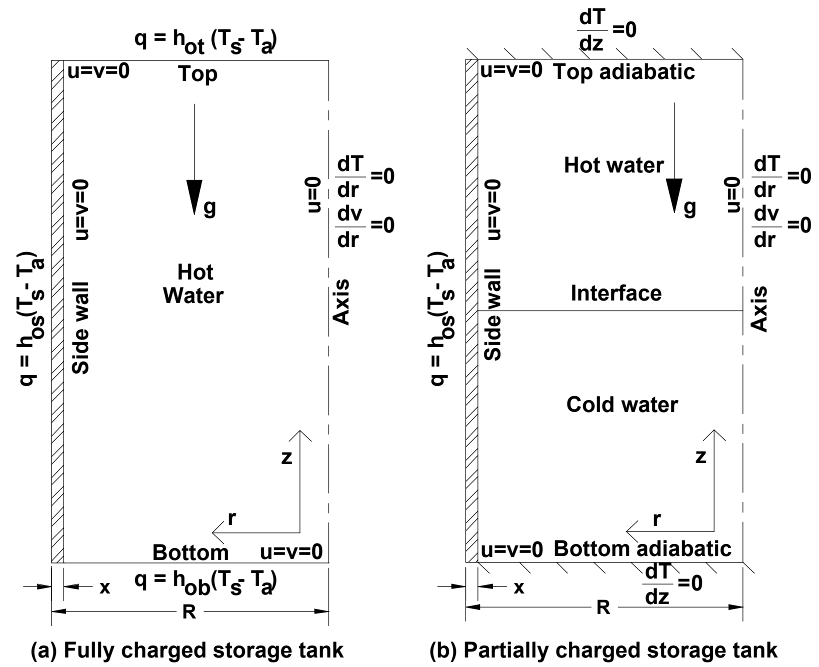

Figure. 1 Physical model and computational domain for hot water storage tank

Polystyrene insulation with a thermal conductivity of 0.035 $Wm^{-1}K^{-1}$ and thickness 15mm is used as insulating material on the exterior tank wall surface. A typical storage tank with aspect ratio of 3 has length 1.62m and diameter 0.54m. The heat transfer from the tank walls to the ambient results in loss of energy in the tank, which is attributed due to turbulent natural convection. Two cases are considered as follows: a) convective heat transfer on all sides (b) convective heat transfer only on side walls while the bottom and top walls are considered adiabatic. The assumptions made in this mathematical formulation are axi-symmetric, incompressible, buoyancy driven transient natural convection flow and negligible viscous dissipation.
The governing equations as given in **Appendix A** are discretised using a computational code [FLUENT]. In fully charged tank turbulent equations are solved whereas for partially charged tank, Volume of Fluid (VOF) model [7] generally suited for two phase flows is considered by taking water properties at hot and cold temperatures separately as two layers. A single set of momentum equations thus obtained is solved throughout the domain, and the resulting velocity field is shared between the layers.

Performance Measures

<u>*Mix Number:*</u>

Mix number is used to estimate the degree of stratification based on the energy distribution inside the tank [8].Thus,

Moment of enthalpy, $m = \sum y_i E_i$ (1)

where $1 < i \leq n$ and n is the number of uniform temperature segments the tank is divided into, y_i is the distance of the i^{th} layer of fluid from bottom of the tank and E_i is the enthalpy given by $E_i=?V_i c_p T_i$. Mix Number is zero when the tank is partially charged and unity for a fully charged tank.

$$\text{Mix Number (M)} = \frac{m_{ideal,stratified} - m_{actual,stratified}}{m_{ideal,stratified} - m_{fully,mixed}} \quad (2)$$

<u>*Exergy efficiency:*</u>

Fully charged and partially charged tank

$$\text{Actual exergy, } E_{actual}=?\left(T\right)=c_p\left[\left(T_{mf/mp}-T_a\right)-T_a\ln\left({T_{mf/mp}}\big/{T_a}\right)\right] \quad (3)$$

The maximum available exergy capacity is calculated as:

$$\text{Maximum available exergy, } E_{max}=?\left(T\right)=c_p\left[\left(T_{initial}-T_a\right)-T_a\ln\left({T_{initial}}\big/{T_a}\right)\right]$$

(4)

$$\text{Exergy efficiency, } ?_{exergy}=\frac{?_{actual}}{?_{max}}$$

Validation

Numerical analysis of transient turbulent natural convection in a vertical cylindrical hot water storage tanks is studied. The present numerical results is validated with published experimental results of Abdoly and Rapp [5] as shown in Figure. 2 and 3. The geometry consists of stainless steel cylindrical tank of aspect ratio 6 (length=1.62m, diameter=0.27m) with initial temperatures as T_1=T_2=364 K and T_3 =288 K. The tank is subjected to convective heat loss to the ambient. In case of fully charged storage tank, temperature profile shows the formation of temperature gradient and partially charged storage tank, the thermal decay with time due to mixing is clearly seen in temperature profile.

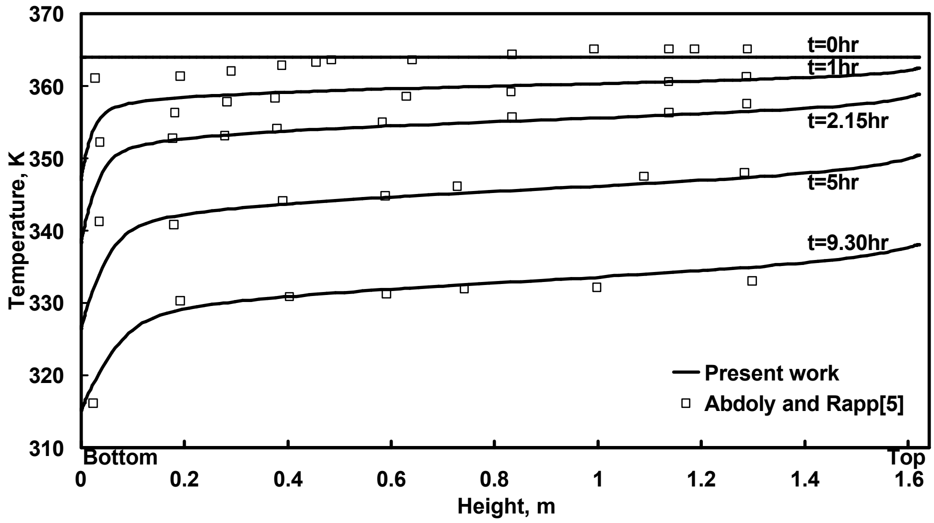

Figure. 2 Validation of present numerical work for fully charged storage tank

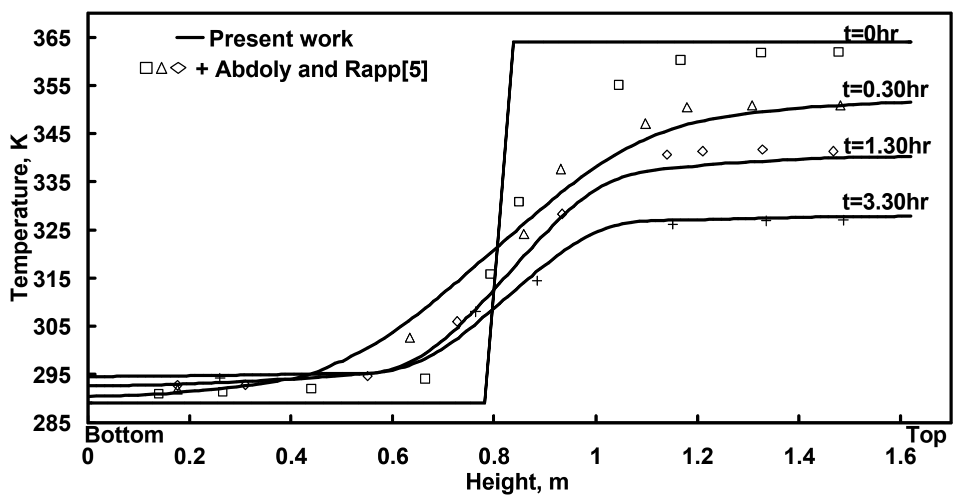

Figure. 3 Validation of present numerical work for partially charged storage tank

Results

Isotherms and velocity contours in Figure.4 shows heat loss to the ambient induces convective

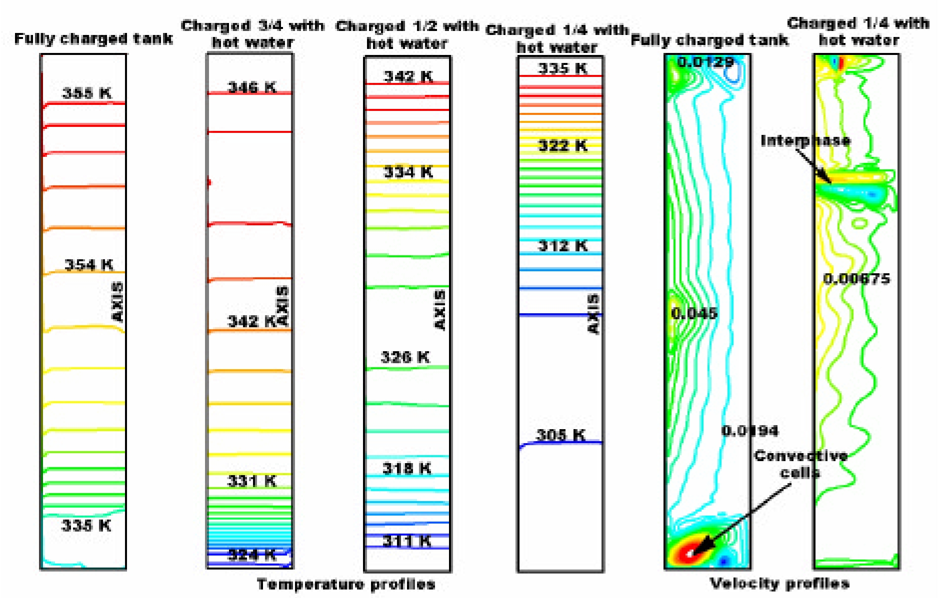

Figure. 4 Temperature/velocity(m/s) profiles for fully charged and partially charged hot water storage tanks (aspect ratio=3 , time=2hr, T_1= T_2= 358 K and T_3= 298 K)

cells near to the inner wall region (boundary layer regime) inducing the downward motion of the fluid near the wall, thereby promoting stratification inside the fully charged tank whereas the same effect induces the decay of stratified layers inside the partially charged tank. The reason for this mixing being convective heat loss to the ambient, axial tank wall conduction and heat diffusion between hot and cold fluids.

Mix Number Analysis:

Mix Numbers plotted in Figures. 5 for fully charged storage tank shows linearly decreasing trend with time. As time increases rate of mixing decreases, as heat loss to ambient decreases. Mixing also decreases with increasing aspect ratio which given by typical values of M=0.829 for aspect ratio 4 and M=0.9102 for aspect ratio 1. It is to be noted that the value of M =1 completely mixed tank.

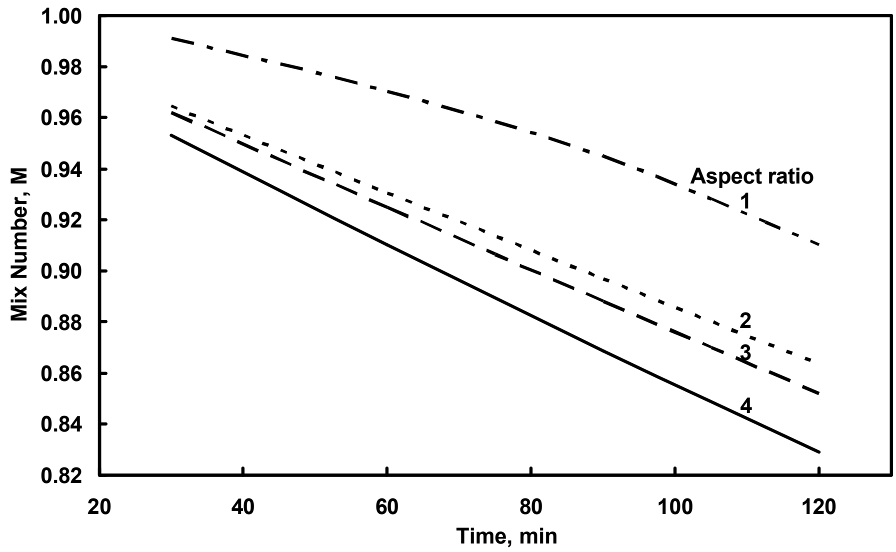

Figure. 5 Mix Number variation for fully charged storage tank

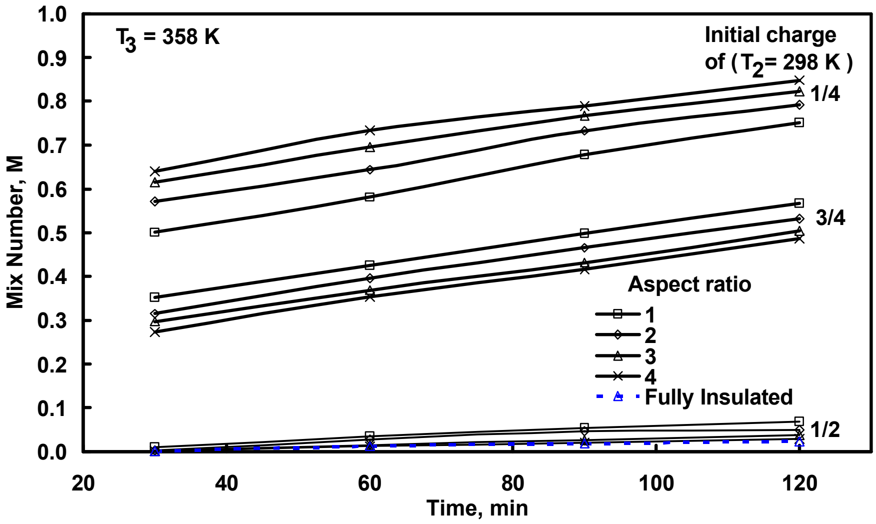

Figure.6 Mix Number variation for partially charged storage tank

Mix Number plotted in Figure. 6 for partially charged storage tank shows for specified time interval, mixing decreases with increase in aspect ratio for partially charged tanks with initial charge of ¾ and ½ as seen in Figure. 6. For partially charged tank with hot water at T_3 charged to ¾ level, higher mixing is observed compared to tanks charged to ¼ and ½ levels. Providing insulation on the outer tank wall decreases heat loss to ambient but the axial wall conduction effects still contributes to mixing as is seen the values of M=0.0236 at time 2hr in Figure.6

<u>*Exergy efficiency:*</u>

The exergy efficiency for higher aspect ratio is more than the lower aspect ratio as tabulated in the *Table. 1* for specified time of 2hrs. The reason is convective heat loss from the tank wall to the ambient by natural convection leads to development of temperature gradient, as a result the stored energy inside the tank is decreased, leading to decrease in the Exergy efficiency. The development of temperature gradients is attributed due to mixing inside the storage tank. It is also seen that for large Rayleigh number tanks development of stratification leads to better efficiency due to less mixing.

Table. 1 Exergy efficiency for fully and partially charged tank of varying aspect ratio

Aspect ratio	Exergy efficiency			
	$T_1 = 358$ K	$T_2 = 358$ K and $T_3 = 298$ K		
	Fully charged	½ charged	¾ charged	¼ charged
1	84.17	95.88	73.07	59.16
2	86.21	96.22	73.98	66.38
3	87.14	96.31	76.01	68.57
4	93.14	96.52	78.43	68.69

Conclusion

A numerical study of transient turbulent natural convection and thermal stratification in fully and partially charged storage tanks in static operation mode presented. Large Rayleigh number tanks shows less mixing. In fully charged case, mixing decreases with increase in aspect ratio which is also seen in partially charged tank. In addition the tank charged to ½ shows less mixing compared to ¾ and ¼ charged tanks. Providing insulation in both cases reduces mixing and increases the Exergy efficiency. Large values of initial temperature differences leads to establishment of perfect thermocline but increases mixing due to buoyancy effects. Natural convective effects are clearly seen in the hot water storage tanks.

Appendix. A Governing Equations and boundary conditions:

Fully charged storage tank

Continuity Equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(v)=0 \quad (A.1)$$

Momentum Equation:

r-direction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=-\frac{1}{?}\frac{\partial p}{\partial r}+?\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right]$$

(A.2) z-direction

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=-\frac{1}{?}\frac{\partial p}{\partial z}+?\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{\partial^2 v}{\partial z^2}\right]-\frac{??}{?}g \quad (A.3)$$

Energy Equation:

$$?c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}\right]=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)\right] \quad (A.4)$$

Partially charged storage tank

Continuity equation:

$$\frac{\partial}{\partial t}(v_f)+u\frac{1}{r}\left(\frac{\partial}{\partial r}(v_f)\right)+v\frac{\partial}{\partial z}(v_f)=0 \quad (A.5)$$

The volume fraction equation will be solved for the primary phase if $v_{f1}+v_{f2}=1$.

Momentum equation:

r-direction

$$\frac{\partial}{\partial t}\left(?_{L1}u_{L1}+?_{L2}u_{L2}\right)+\frac{\partial}{\partial r}\left(?_{L1}u_{L1}^2+?_{L2}u_{L2}^2\right)+\frac{\partial}{\partial z}\left(?_{L1}u_{L1}v_{L1}+?_{L2}u_{L2}v_{L2}\right)=$$
$$-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(u_{L1}+u_{L2}\right)\right)+\frac{\partial}{\partial^2 z}\left(u_{L1}^2+u_{L2}^2\right)-\frac{\left(u_{L1}+u_{L2}\right)}{r^2}\right]$$

(A.6)

z-direction

$$\frac{\partial}{\partial t}\left(?_{L1}v_{L1}+?_{L2}v_{L2}\right)+\frac{\partial}{\partial r}\left(?_{L1}u_{L1}v_{L1}+?_{L2}u_{L2}v_{L2}\right)+\frac{\partial}{\partial z}\left(?_{L1}v_{L1}^2+?_{L2}v_{L2}^2\right)=$$
$$-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(v_{L1}+v_{L2}\right)\right)+\frac{\partial}{\partial^2 z}\left(v_{L1}^2+v_{L2}^2\right)-\frac{\left(v_{L1}+v_{L2}\right)}{r^2}\right]+\left(\frac{??_{L1}}{?_{L1}}+\frac{??_{L2}}{?_{L2}}\right)g \qquad \text{(A.7)}$$

where $?_{L1}=a?_1$, $?_{L2}=\left(1-a\right)?_1$ and $a=\frac{v_{f1}}{v_{f1}+v_{f2}}$

The effect of volume fractions will be taken into account by

$?=a?_{L1}+(1-a)?_{L2}$: $u=au_{L1}+(1-a)u_{L2}$

Enthalpy equation:

$$\frac{\partial}{\partial t}\left(?_{L1}H_{L1}+?_{L2}H_{L2}\right)+\frac{\partial}{\partial r}\left(?_{L1}u_{L1}H_{L1}+?_{L2}u_{L2}H_{L2}\right)+\frac{\partial}{\partial z}\left(?_{L1}v_{L1}H_{L1}+?_{L2}v_{L2}H_{L2}\right)$$
$$=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)+\left(S_{L1}^{'}+S_{L2}^{'}\right) \qquad \text{(A.8)}$$

where “L1” first layer and “L2” indicates the second layer.

Transport equation for k and e

The standard k-e turbulence model [1] is used for the turbulent flow calculations for Ra > 10^9 conditions.

$$\frac{\partial}{\partial t}(?k)+\mathrm{div}(?k\mathrm{u})=\mathrm{div}\left[\left(\mu+\frac{\mu_{tur}}{s_k}\right)\mathrm{grad}(k)\right]+E_k+E_b-?e$$

(A.9)

$$\frac{\partial(?e)}{\partial t}+\mathrm{div}(?e\mathrm{u})=\mathrm{div}\left[\left(\mu+\frac{\mu_{tur}}{s_e}\right)\mathrm{grad}e\right]+C_1\frac{e}{k}\mu_{tur}\left(E_k+C_3E_b\right)-C_2?\frac{e^2}{k}$$

(A.10)

In these equations, E_k and E_b represent the generation of turbulent kinetic energy due to the mean velocity gradients and due to buoyancy respectively. C_μ,C_1,C_2,C_3 are model constants, whereas σ_k and σ_ε are the turbulent Prandtl numbers for k and ε respectively. The model constants have the following values:

$C_\mu=0.09$; $s_k=1.00$; $s_e=1.30$; $C_1=1.44$; $C_2=1.92$

(A.11)

The effective turbulent viscosity is computed by combining k and ε as given by [1]:

$$\mu_{tur}=?c_\mu\frac{k^2}{e}$$

(A.12)

The initial and boundary conditions are shown in *Table. 2* and *Table. 3* respectively. In the above formulation, the ambient temperature T_a is taken to be 303 K.

The outer surface heat transfer coefficients on the sidewall and top/bottom wall are computed from local Nusselt number correlation for free convection [6]:

$$Nu = \frac{h_{ow} L}{k_f} = c\left(GrPr\right)^m \tag{A.13}$$

The values for c and m are obtained from *Table. 4*

Table. 1 Initial conditions for fully and partially charged storage tanks

Fully mixed storage tank	u(r,z) = 0; v(r,z) = 0; p(r, z) = p; $T(r, z) = T_1$
Stratified storage tank	u(r,z) = 0; v(r,z) = 0; p(r, z) = p; $T(r,z) = T_2;\ 0 < z \le nL$ $T(r,z) = T_3;\ nL < z \le L$ n = ¾ , ½ , ¼

Table. 2 Boundary conditions for fully and partially charged storage tanks

	Convection on all sides	**Convection on side wall only**
Top	$-k\frac{\partial T}{\partial x} = h_{ot}(T - T_a)$	$\frac{\partial T}{\partial z} = 0$
Bottom	$-k\frac{\partial T}{\partial x} = h_{ob}(T - T_a)$	$\frac{\partial T}{\partial z} = 0$
Side	$-k\frac{\partial T}{\partial x} = h_{os}(T - T_a)$	$-k\frac{\partial T}{\partial x} = h_{os}(T - T_a)$

Table. 4 Values of constants c and m in Eq. (A.13)

Configuration	c	M	GrPr
Vertical [sidewall]	0.10	0.333	$> 10^9$
Horizontal [Top/Bottom wall]	0.15	0.333	$> 10^9$

References:

[1] **Hsieh KJ, Lien FS.** Numerical modeling of buoyancy driven turbulent flows in Enclosures. 2004; (Article in Press)

[2] **Bouhdjar A, Harhad A.** Numerical analysis of transient mixed convection flows in storage tank: influence of fluid properties and aspect ratios on stratification. Renewable Energy 2002; 25: 555-567.

[3] **Nelson J.E.B., Balakrishnan A.R., Srinivasa Murthy S.,** Experiments on stratified chilled water tanks, Int J of Refrigeration 1999; 22: 216-234.

[4] **Shyu RJ, Lin JY, Fang LJ.** Thermal Analysis of Stratified Storage Tanks. ASME Trans. 1989; 111:54-61.

[5] **Abdoly MA, Rapp D.** Theoretical and experimental studies of stratified thermocline storage of hot water. Energy Convers. Mgmt 1982; 22: 275-285.

[6] **Churchill S.W., Chu H.S.H.,** Correlating equations for laminar and turbulent free convection from a vertical plate, Int. J. Heat Mass Transfer 1975; 18:1323-1329.

[7] **Patankar S. V.,** Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corp., Washington DC, 1980.

[8] **Dincer I, Marc A.Rosen.** Thermal Energy Storage Systems and Applications, Wiley and Sons Ltd. England , 2002